Interatomic Potential of Li–Mn–O and Molecular Dynamics Simulations on Li Diffusion in Spinel Li$_{1-x}$Mn$_2$O$_4$

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ABSTRACT: An interatomic potential of the Li–Mn–O ternary system has been developed on the basis of the second-nearest-neighbor modified embedded-atom method (2NN MEAM) formalism combined with a charge equilibration (Qeq) concept. The potential reproduces fundamental physical properties (structural, elastic, thermodynamic and migration properties) of various compounds well, including lithium oxides, manganese oxides, and lithium manganese ternary oxides. Through molecular dynamics (MD) simulations using the developed potential, lithium diffusion properties (activation energy for lithium migration and diffusion coefficient) in spinel Li$_{1-x}$Mn$_2$O$_4$ are also reproduced in good agreement with experiments. We have found that the effect of the lithium vacancy concentration is marginal on the activation energy for lithium diffusion in the Li$_{1-x}$Mn$_2$O$_4$ cathode, but it is significant in the lithium diffusion coefficient. The potential can be further utilized for atomistic simulations of various materials phenomena (phase transitions, defect formation, lithiation/delithiation, etc.) in LIB cathode materials.

1. INTRODUCTION

From the early stages of battery development, LiCoO$_2$ has been the most widely used cathode material for lithium ion batteries (LIB) due to its characteristics of high voltage, fast Li ion migration, and high capacity.† Despite the successful commercialization of LiCoO$_2$ as a cathode material over the past 20 years, searches for a replacement material with low Co content have been extensively conducted due to high materials cost, toxicity of Co, and performance degradation during overcharge.‡,§ The number of possible compounds and doping elements is almost limitless, making it laborious and inefficient to search for new cathode materials. Moreover, important materials phenomena of cathode materials, such as diffusion, phase transition, defect formation, and lithiation/delithiation, are atomic-scale behavior that is difficult to observe experimentally. Therefore, in order to develop LIB cathode materials, it is necessary to have an effective and reliable approach to investigate such materials phenomena. In this case, computational approaches can provide useful insight and guidance into materials phenomena that are rather difficult to examine directly through experimental methods. Among such approaches, first-principles density functional theory (DFT) calculations provide the most reliable information on (sub-)atomic-scale physical properties. However, due to limits of size (or the number of atoms), it is often not possible to investigate material behaviors using only DFT calculations. Another approach is large-scale atomistic simulation using a (semi-)empirical interatomic potential model. This can be an effective method to investigate the aforementioned materials phenomena for LIB cathode materials on a larger scale.

However, even with the clear necessity of large-scale atomistic simulation, relatively few efforts have been made for the LIB cathode materials. We believe that this is not because the atomistic simulation is not important, but because it has been difficult to develop suitable interatomic potential models (and parametrization) for the LIB cathode materials. The materials system for LIB cathode materials consists mainly of multicomponent lithium transition-metal oxides (Li–TM–O). Therefore, the potential model should be able to simultaneously cover various bonding natures, such as metallic, covalent, and ionic bonds in the material under consideration. In addition, the model should be able to describe realistically that the charge state of atoms can vary dynamically during the charge/discharge process due to changes in crystal structure, lithium concentration, and oxidation states of transition metals, etc. For these reasons, a many-body potential coupled with a variable charge model is appropriate for covering the complicated LIB cathode materials system. Most previous studies, however, have used a pairwise-type potential with a fixed or partially variable charge model.†–‡ The classical pairwise potential model is not suitable to cover the wide range of materials systems associated with LIB cathode materials, and
the fixed charge model only describes ionic bonds for fixed charge states in a simplified method. Therefore, previous studies\textsuperscript{1}–\textsuperscript{12} have been restricted only to a relatively simple modeling of specific materials (or structures).

For a practical modeling of a wider range of LIB cathode materials, many-body potentials coupled with the variable charge model are more appropriate, as mentioned earlier. For the variable charge model, the charge equilibration (Qeq) method developed by Rappe and Goddard\textsuperscript{13} has been effectively used. Depending on how the many-body part is handled, one can think of a few candidate potential models: the ReaxFF\textsuperscript{14}–\textsuperscript{16} the charge-optimized many-body (COMB) potential\textsuperscript{17,18} and the combination of a modified embedded atom method (MEAM) with Qeq. Islam et al.\textsuperscript{19} developed a reactive force field (ReaxFF) potential for the Li–S system and, using molecular dynamics (MD) simulations, provided insight into the behavior of sulfur-based cathode materials that are needed to develop lithium–sulfur batteries. To the best of our knowledge, there is no application of the ReaxFF and the COMB potential to lithium metal oxide systems. This is probably due to the fact that the formalism for ReaxFF and the COMB potential has been designed and applied primarily for covalent materials rather than metallic compounds. On the other hand, the combination of MEAM and Qeq can be more advantageous in developing interatomic potential for the lithium metal oxide systems, because it is well-known that MEAM is a robust potential model for metallic compounds. Nevertheless, there are still very few studies using this kind of potential model for LIB cathode materials. Kong et al.\textsuperscript{19} developed a charge-transfer (CT) MEAM potential for the ternary Li–Mn–O system and predicted fundamental materials properties such as the charge states and redox potentials of Li\textsubscript{2}MnO\textsubscript{3}. Even though they report a wide range of materials properties calculated at 0 K, they do not report dynamic properties obtained using MD simulations at finite temperatures. Further, the applicability of the potential in subunary and binary systems has not been reported in sufficient detail. The applicability of an interatomic potential at finite temperatures is decisive for practical application of the potential to an investigation of dynamic behavior of materials. The applicability to subunary and binary systems is also important for robust applications of the potential to a wide range of materials phenomena, for example, the lithiation of manganese oxides.

In a situation in which a reliable interatomic potential is not well prepared, it is obviously necessary to develop a new interatomic potential covering multicomponent metal oxide systems in order to enable large-scale atomistic simulations for LIB cathode materials. The second-nearest-neighbor (2NN) MEAM,\textsuperscript{21,22} one of the latest versions of MEAM,\textsuperscript{20,23} has been applied to a wide range of materials, including metals, covalent elements, and their alloys. Especially, it has been successfully applied to essential metallic elements associated with LIB cathode materials, such as Li,\textsuperscript{24} Co,\textsuperscript{27} Mn,\textsuperscript{25} Ni,\textsuperscript{29} Al\textsuperscript{30} and some of their alloys. Recently, our group further extended the 2NNMEAM to cover multicomponent oxide compounds by combining it with the Qeq\textsuperscript{13} concept (2NNMEAM+Qeq).\textsuperscript{25} Special attention to the removal of known problems found in the original Qeq method has been considered in the implementation of the Qeq method to the 2NNMEAM. From this point of view, we believe that 2NNMEAM+Qeq can be one of the most suitable potential models for LIB cathode materials systems.

An interatomic potential database for relevant materials needs to be developed to investigate materials phenomena (diffusion, phase transition, defect formation, lithiation/delithiation, etc.) at the atomic-scale and to achieve the ultimate goal of searching for optimum materials. As a part of a long-term project to develop an interatomic potential database for LIB cathode materials systems, the objective of this study is to develop a potential for the ternary Li–Mn–O system (including binary Li–O, Mn–O, and Li–Mn systems) based on the 2NNMEAM+Qeq potential formalism. In addition, we also carry out Li diffusion simulations to confirm that the dynamic material phenomena in LIB cathode materials can be reliably described through MD simulation using the developed potential. A brief description of the formalism and the parameters of the potential model (2NNMEAM+Qeq) is given in section 2. Then, we report on the quality of the parametrization for the ternary Li–Mn–O system as well as for the binary Li–O, Mn–O, and Li–Mn systems in section 3, paying attention to the stability of the potentials at finite temperatures. Finally, in section 4, we show that MD simulation using the developed potential can be a powerful and effective tool to evaluate diffusion properties in LIB cathode materials.

## 2. 2NNMEAM+Qeq Potential Formalism and Parameters

In the 2NNMEAM+Qeq potential formalism,\textsuperscript{20} the total energy of a system contains two energy terms—nonelectrostatic and electrostatic energy terms—as expressed in eq 1. The nonelectrostatic term is exactly identical to the existing 2NNMEAM potential formalism\textsuperscript{26} and is independent from the electrostatic term (charges). The electrostatic term is a function of the atomic positions ($\mathbf{r}$) and charges ($\mathbf{q}$).

$$E^{\text{Total}} = E^{\text{MEAM}}(\mathbf{r}) + E^{\text{ES}}(\mathbf{r}, \mathbf{q})$$

(1)

The MEAM energy is approximated as

$$E^{\text{MEAM}} = \sum_i \left[ F(\bar{\rho}_i) + \frac{1}{2} \sum_{j \neq i} S_{ij} \phi_{ij}(R_{ij}) \right]$$

(2)

where $F_i$ is the embedding function, $\rho_i$ is the background electron density at site $i$, $S_{ij}$ and $\phi_{ij}(R_{ij})$ are the screening function and the pair interaction between atoms $i$ and $j$ separated by a distance $R_{ij}$, respectively. The details in the expression of MEAM energy are beyond the scope of this paper, so readers are referred to the literature\textsuperscript{21,22} or to the Supporting Information (SI) for more details.

The electrostatic energy is expressed as the sum of atomic energy $E^{\text{atom}}_{\text{Coul}}$ (or penalty energy) and the Coulomb pair interaction $V_{\text{Coul}}^{\text{Coul}}$.

$$E^{\text{ES}} = \sum_i E^{\text{atom}}_{\text{Coul}}(q_i) + \sum_{ij \neq i} \frac{1}{2} V_{\text{Coul}}^{\text{Coul}}(q_i, q_j, R_{ij})$$

(3)

where $q_i$ is the charge state of atom $i$. The charge state of each atom depends on the local environment. According to the Qeq concept,\textsuperscript{13} the equilibrium charge can be computed by minimizing the total electrostatic energy (eq 3) under the condition of charge conservation. The original Qeq uses a simple quadratic function for the atomic energy. This is probably because one can solve the minimization problem simply via a linear method when using the simple quadratic function. However, this often leads to the instability of charge
The Coulomb pair interaction is computed through the $q_i$ that the conventional atom-based charge description, $q_i$, and Mn$^{2+}$ without any modification by $\alpha$. Therefore, we optimized by fitting the fundamental physical properties of lithium oxides and manganese oxides. For a ternary system, six more parameters for the $i$--$k$--$j$ triplet $[C_{\text{min}}(i-k-j), C_{\text{max}}(i-k-j), C_{\text{min}}(i-j-k), C_{\text{max}}(i-j-k), C_{\text{min}}(j-i-k), C_{\text{max}}(j-i-k), \rho_i(j)/\rho_i(j)]$ determine the screening effect of the $k$ atom between the $i$--$j$ pair. Therefore, a total of sixty-two parameters (7 Qeq parameters each for Li and Mn; 14 binary MEAM parameters each for Li--Mn, Li--O, and Mn--O; and 6 ternary MEAM parameters for Li--Mn--O) were optimized in this study.

To complete the ternary Li--Mn--O potential parameter set, we first optimize the Qeq parameters of the metal elements (Li and Mn) and the 2NNMEAM parameters of the metal--oxygen pairs (Li--O and Mn--O) by fitting the properties of each binary metal oxide system. After determining the Li--O and Mn--O binary potential parameters, we finally optimize the 2NNMEAM parameters for the binary Li--Mn and ternary Li--Mn--O systems by fitting the lithium manganese oxide properties. Since the properties of oxide systems are determined by combined effects of the MEAM and Qeq parts, the parameters from the two parts are optimized simultaneously. In case of the Li--Mn binary system, there is no stable compound to be considered for parameter optimization. Therefore, we first determine the $E_o$, $R_o$, $\alpha$, $d_{\text{rep}}$, $d_{\text{att}}$ parameters of the Li--Mn pair by fitting the equation of state (EOS) for a given reference structure (B2 in this study) to DFT calculation. Then, the remaining 2NNMEAM parameters for the Li--Mn pair and those for the Li--Mn--O triplet are optimized by fitting the fundamental physical properties of the
Calculation and Average Atomic Charges (migration energy barrier in anti-potential for di-type LiMn$_2$O$_4$. The redox potential, which describes the voltage point defect formation energy are calculated and compared well. In addition, defect properties such as surface energy and energetics of the charge/discharge process in cathode materials needed to extract lithium atoms out of the bulk system, is 41.

To obtain reasonable insight from the results of atomistic simulations, the interatomic potential should be accurate and well-verified for the system under consideration. Basically, a reliable interatomic potential should be able to correctly reproduce various fundamental physical properties (structural, elastic, thermodynamics, etc.) of the relevant materials. We calculate the fundamental physical properties of the Li–Mn, Li–O, Mn–O, and Li–Mn–O oxide compounds using the developed potentials and compare them with relevant experimental data and/or DFT calculations to evaluate the quality of the potential. To ensure the reproducibility of the potential for diffusion properties, we calculate the lithium migration energy barrier in anti-fluorite-type Li$_2$O and spinel-type LiMn$_2$O$_4$. The redox potential, which describes the voltage needed to extract lithium atoms out of the bulk system, is calculated to confirm that our potential reproduces the energetics of the charge/discharge process in cathode materials well. In addition, defect properties such as surface energy and point defect formation energy are calculated and compared with those from DFT calculations. We do not include the evaluation for Li–Mn, Li–O, and Mn–O binary systems in this section but present them in the SI and cover only the Li–Mn–O ternary system in this section.

For calculations, we use our own in-house code in which the 2NNMEAM+Qeq formalism is implemented. The code is provided as part of the SI of this paper. To obtain an equilibrium structure of various phases, the internal stresses in the initial samples are completely relaxed by allowing atomic movement and variation of cell size. At the initial step of the simulation, the equilibrium charges of individual atoms are assigned by performing the Qeq. The Qeq process is performed at every 1000 steps rather than at every single step because it requires more expensive computational costs than do conventional energy calculations. We have confirmed that the calculation results are not significantly dependent on the frequency of the Qeq process. The cutoff distances for 2NNMEAM and Coulomb interaction are taken as 4.8 and 12.0 Å, respectively.

3. EVALUATION OF THE DEVELOPED POTENTIALS

Table 2. Lattice Parameters of Lithium Manganese Oxide Phases in Comparison with Experimental Data and DFT Calculation

<table>
<thead>
<tr>
<th>phase</th>
<th>2NNMEAM+Qeq</th>
<th>% error</th>
<th>expt</th>
<th>DFT$^a$</th>
<th>% error</th>
</tr>
</thead>
<tbody>
<tr>
<td>spinel LiMn$_2$O$_4$</td>
<td>a (Å)</td>
<td>8.0669</td>
<td>−2.19</td>
<td>8.2474$^b$, 8.2483$^b$</td>
<td>8.431</td>
</tr>
<tr>
<td>layered Li$_2$MnO$_3$</td>
<td>a (Å)</td>
<td>4.7492</td>
<td>−3.73</td>
<td>4.9292$^c$, 4.9377$^c$</td>
<td>5.011</td>
</tr>
<tr>
<td>b (Å)</td>
<td>8.5646</td>
<td>−0.88</td>
<td>8.5315$^d$, 8.532$^d$</td>
<td>8.633</td>
<td>+1.19</td>
</tr>
<tr>
<td>c (Å)</td>
<td>4.7244</td>
<td>−6.03</td>
<td>5.0251$^e$, 5.030$^e$</td>
<td>5.092</td>
<td>+1.28</td>
</tr>
<tr>
<td>β (deg)</td>
<td>109.5610</td>
<td>+0.15</td>
<td>109.337$^f$, 109.464$^f$</td>
<td>109.48</td>
<td>+0.07</td>
</tr>
<tr>
<td>layered LiMnO$_2$</td>
<td>a (Å)</td>
<td>4.7276</td>
<td>−13.02</td>
<td>5.431$^g$, 5.439$^g$</td>
<td>5.488</td>
</tr>
<tr>
<td>b (Å)</td>
<td>2.7925</td>
<td>−2.83</td>
<td>2.809$^h$, 2.809$^h$</td>
<td>2.872</td>
<td>+2.24</td>
</tr>
<tr>
<td>c (Å)</td>
<td>4.9530</td>
<td>−8.15</td>
<td>5.390$^i$, 5.395$^i$</td>
<td>5.395</td>
<td>+0.05</td>
</tr>
<tr>
<td>β (deg)</td>
<td>108.5520</td>
<td>−6.36</td>
<td>115.95$^j$, 115.9$^k$</td>
<td>115.412</td>
<td>−0.44</td>
</tr>
<tr>
<td>o-LiMnO$_2$</td>
<td>a (Å)</td>
<td>3.9093</td>
<td>−14.58</td>
<td>4.574$^l$, 4.5795$^l$</td>
<td>4.634</td>
</tr>
<tr>
<td>b (Å)</td>
<td>5.6825</td>
<td>−1.41</td>
<td>5.752$^m$, 5.7750$^m$</td>
<td>5.833</td>
<td>+1.21</td>
</tr>
<tr>
<td>c (Å)</td>
<td>2.7342</td>
<td>−2.67</td>
<td>2.808$^n$, 2.8106$^n$</td>
<td>2.869</td>
<td>+2.13</td>
</tr>
<tr>
<td>t-LiMnO$_2$</td>
<td>a (Å)</td>
<td>5.5688</td>
<td>−1.43</td>
<td>5.6488$^o$, 5.6504$^o$</td>
<td>5.787</td>
</tr>
<tr>
<td>rRMSE (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>6.62</td>
</tr>
</tbody>
</table>

$^a$Reference 32. $^b$Reference 33. $^c$Reference 34. $^d$Reference 35. $^e$Reference 36. $^f$Reference 37. $^g$Reference 38. $^h$Reference 39. $^i$Reference 40. $^j$Reference 41. $^k$Reference 42.

Table 3. Bulk Modulus (B) and Enthalpy of Formation ($\Delta H_f$) of Lithium Manganese Oxide Phases in Comparison with DFT Calculation and Average Atomic Charges (q$_{avg}$) Assigned to Li, Mn, and O Atoms in Each Phase

<table>
<thead>
<tr>
<th>phase</th>
<th>B (GPa)</th>
<th>$\Delta H_f$ (eV/atom)</th>
<th>q$_{avg}$ (e)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2NNMEAM+Qeq</td>
<td>DFT$^a$</td>
<td>2NNMEAM+Qeq</td>
</tr>
<tr>
<td>spinel LiMn$_2$O$_4$</td>
<td>147</td>
<td>112</td>
<td>−1.943</td>
</tr>
<tr>
<td>layered Li$_2$MnO$_3$</td>
<td>182</td>
<td>113</td>
<td>−2.087</td>
</tr>
<tr>
<td>layered LiMnO$_2$</td>
<td>217</td>
<td>112</td>
<td>−2.115</td>
</tr>
<tr>
<td>o-LiMnO$_2$</td>
<td>188</td>
<td>111</td>
<td>−2.114</td>
</tr>
<tr>
<td>t-LiMnO$_2$</td>
<td>150</td>
<td>113</td>
<td>−2.095</td>
</tr>
</tbody>
</table>

$^a$Reference 42.
migration energy barrier, redox potential, surface energy, and point defect formation energy. We also check the stability of the ternary compounds in MD simulations over a wide temperature range. Table 2 shows the calculated lattice parameters of lithium manganese oxide compounds in comparison with experimental data32–41 and DFT calculation.42 The present potential generally underestimates the lattice parameters compared to the experiments, and the relative root-mean-squared error (rRMSE) is 6.62%. The error is mainly generated from LiMnO2 phases because more weight is given to the structural properties of spinel LiMnO4 and layered Li2MnO3, which are more easily synthesized in experiments than are LiMnO2 phases. Table 3 summarizes the calculated bulk modulus, enthalpy of formation, and average atomic charges in comparison with those from DFT calculation.42 The present potential generally overestimates the bulk modulus compared to the DFT calculation. The calculated enthalpy of formation agrees well with that from the DFT calculation. The charge states of Li, Mn, and O in our calculation are somewhat lower than the formal charges. We do not consider the charge states of individual elements as a target property for fitting, but we obtain them as a result of the fitting to better reproduce other properties and to make the simulation stable. Too large charge values often cause the instability problem during the simulation. The energy contribution from the electrostatic Qeq part in the total energy of lithium manganese oxide system is about 60%, indicating that the charge transfer effect is sufficiently considered in our calculation. The properties mentioned next are calculated from samples including defects (thermal fluctuations and vacancies), and the charge states of individual atoms are automatically determined by the Qeq, depending on their neighboring environments. The electrostatic neutrality of the system is always maintained.

The diffusion properties of lithium have an important influence on the performance of LIB cathode materials. One of the main purposes of the present potential development for the Li–Mn–O system is to investigate the lithium diffusion properties in cathodes through large-scale atomistic simulations at finite temperatures. As a means to see if the present potential can predict the diffusion properties reasonably well through MD simulations, we calculate the lithium migration energy barrier in spinel LiMnO4 at 0 K, in advance. In the spinel MD simulations, we calculate the lithium migration energy barrier to be around 0.7 eV, and 0.54 eV (for the whole temperature region, 300–800 °C). The saddle shape of the activation energy curve is also reported by Verhoeven et al.,44 with an activation energy value of 0.5 eV for jumping from an 8a to a 16c site, which is comparable to the low-temperature value of Takai et al. Xu and Meng45 investigate the lithium migration energy barrier in spinel LiMnO4 through a DFT calculation under different valence states of Mn ions and their arrangement, and they report calculated values within the range of 0.2–0.8 eV. Hoang46 also reports the lithium migration energy barrier for different neighboring Mn charge states, in the range of 0.19–0.57 eV. According to our calculation, the charge state of all neighboring Mn atoms in the sample are the same due to the identical atomic environment of individual Mn sites. The present calculation for the lithium migration energy barrier agrees reasonably well with the reference data in the absolute value and also in the saddle shape of the energy curve.

To check whether the present potential reproduces the redox reactions well during the charge/discharge process, we calculate the redox potential and compare it with the DFT calculations. We do not include the redox potential in the target property for the parameter optimization, but we intend to confirm that the parameter set fitted to the enthalpy of formation of lithium and oxides reproduces the redox potential well. The redox potential describes the voltage needed to extract Li atoms out of a bulk system. It can be computed using

$$V = \frac{G(Li_{x}MO) - G(Li_{x}MO) - (x_2 - x_1)G(Li)}{(x_2 - x_1)e}$$

where $G(X)$ is the Gibbs free energy of $X$, $e$ is the elementary charge, and MO refers to the MnO2 of the lithium manganese oxide compound. The Gibbs free energy can be replaced by the potential energy at 0 K, approximating the contribution of enthalpy (PV term) and entropy (TS term) to be negligible. Table 4 summarizes the calculated redox potential in comparison with experimental data47,48 and DFT calculations49,50 [generalized gradient approximation (GGA) and GGA+U]. It should be mentioned here that we calculated the redox potential for a wide range of Li compositions, for spinel LiMnO4 (0 < x < 1) and layered Li2MnO3 (1 < x < 2). The redox potential is almost independent of the lithium composition, and only the average values are presented in Table 4. For spinel LiMnO4, the calculated redox potential associated with full delithiation (LiMn2O4/2 – MnO2), 3.47 V,
somewhat underestimates the experimental value (4.1 V)\textsuperscript{47} and the GGA+U calculation (4.04 V),\textsuperscript{46} while being rather close to the GGA calculation (3.37 V).\textsuperscript{59} Because Li ions can be extracted from different lithium sites (4h, 2c, and 2b) in the layered Li$_2$MnO$_3$ structure, the redox potential for layered Li$_2$MnO$_3$ is calculated from several configurations of delithiated Li$_2$MnO$_3$ according to the lithium vacancy distribution. We obtain the redox potential for layered Li$_2$MnO$_3$/LiMnO$_3$ in the range from 3.58 to 3.80 V. Experimentally, Li$_2$MnO$_3$ is believed to be responsible for the voltage range from 4.4 to 5.0 V.\textsuperscript{48} The redox potential values from DFT calculations using GGA and GGA+U are 4.0–4.2 and 4.5–4.9 V, respectively.\textsuperscript{48} As in the case of spinel Li$_2$MnO$_3$, the present potential underestimates the redox potential of the layered Li$_2$MnO$_3$ compared to the experimental value. GGA also generally underestimates the redox potential but can reproduce the value close to that of the experimental data.\textsuperscript{47} The difference in the redox potential between our calculation and GGA+U comes mainly from the difference in the potential energy of pure lithium between the calculation methods. We think it is meaningless to exactly reproduce the redox potential from GGA+U calculation methods. We think it is meaningless to exactly reproduce the redox potential from GGA+U calculation methods.

We also check whether the present potential reproduces the defect properties such as surface energy and point defect formation energy well. There are various surface structures of spinel Li$_2$MnO$_3$ depending on the orientation and termination. We prepare slab samples with the same surface structure on both sides by moving some of surface ions to the opposite surface, as described in the literature on DFT calculation.\textsuperscript{52–54} We consider seven surface structures for the calculation of the surface energy, and Table 5 shows the calculated surface energies in comparison with the DFT calculations.\textsuperscript{52–54} The present potential reproduces the surface energy and relative stability among surfaces of various structures fairly well. In order to calculate the point defect formation energy of spinel Li$_2$MnO$_3$, we consider point defects where the chemical composition formula does not change, that is, Frenkel and Schottky defects. The X Frenkel defect includes an X vacancy and an X interstitial defect (in a 16c site for the spinel structure), and we calculate the formation energy of the Frenkel defect depending on the relative distance between the vacancy and the interstitial defect (isolated or neighboring). The Schottky defect is the vacancies in stoichiometric units maintaining an overall neutral charge. We consider the Shottky defect by removing one Li, two Mn, and four O atoms in the spinel Li$_2$MnO$_3$ sample containing about 3000 atoms. Since there are many cases in selecting atoms to be removed, we select the atoms randomly and calculate the average energy from multiple trials. The calculated point defect formation energies are shown in Table 6, in comparison with another empirical model (the Born + shell model)\textsuperscript{3} and a DFT calculation.\textsuperscript{46} To the best of our knowledge, there is only one DFT calculation for the Li Frenkel. The DFT calculation is for the second-nearest-neighboring defect pair, which has a value similar to that of the isolated case according to our calculation. Our calculation is comparable to the DFT data\textsuperscript{46} and also the other empirical data\textsuperscript{1} for the Li Frenkel, while there are some discrepancies between our calculation and the other empirical calculation\textsuperscript{1} for the other types of point defect.

According to our experience, so many potentials that perform well at 0 K often fail at finite temperatures. The representative example of this failure is a transformation of the structure into an unknown structure, with a decrease in the energy to a level that makes the unknown structure a stable phase on the phase diagram. In this case, one cannot use the potential for finite-temperature simulations, at least at the relevant compositional region. To check the applicability of the potential for finite-temperature simulations, we examine the energy and structural changes of all compound phases considered during heating and after cooling. The initial structure of each compound phase relaxed at 0 K is heated to 3000 K, increasing the temperature by 200 K and equilibrating the structure (containing 2000–4000 atoms) for 10 ps at each temperature with a Parrinello–Rahman NpT ensemble. Then, the structure heated to each temperature and rapidly cooled to 0 K is examined to determine whether the initial structure and energy have been recovered. From the recovery of the initial (0 K) structure and energy after rapid cooling, one can confirm that the potential does not generate undesirable structural changes during dynamic simulations (at finite temperatures). Figure 2 shows the change in internal energy of lithium

### Table 4. Redox Potentials (V) in Spinel LiMn$_2$O$_4$ and Layered Li$_3$MnO$_3$ in Comparison with Experimental Data and GGA and GGA+U Calculations

<table>
<thead>
<tr>
<th>phase</th>
<th>2NNMEAM $+Q_{eq}$</th>
<th>exp</th>
<th>GGA</th>
<th>GGA+U</th>
</tr>
</thead>
<tbody>
<tr>
<td>spinel LiMn$_2$O$_4$/LiMnO$_3$</td>
<td>3.47</td>
<td>4.1\textsuperscript{e}</td>
<td>3.37</td>
<td>4.04</td>
</tr>
<tr>
<td>layered Li$_3$MnO$_3$/LiMnO$_3$</td>
<td>3.58–3.80</td>
<td>4.0–4.2\textsuperscript{d}</td>
<td>4.5–4.9\textsuperscript{d}</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a}Reference 47. \textsuperscript{b}Reference 48. \textsuperscript{c}Reference 49. \textsuperscript{d}Reference 50.

### Table 5. Surface Energy (J/m$^2$) of Spinel LiMn$_2$O$_4$ for Various Surface Structures in Comparison with DFT Calculations

<table>
<thead>
<tr>
<th>surface structure (orientation—termination)</th>
<th>2NNMEAM $+Q_{eq}$</th>
<th>ref52</th>
<th>ref53</th>
<th>ref54</th>
</tr>
</thead>
<tbody>
<tr>
<td>(001)–Li$_2$</td>
<td>0.58</td>
<td>0.58</td>
<td>0.87</td>
<td>0.554</td>
</tr>
<tr>
<td>(001)–Mn$_3$O$_6$</td>
<td>0.93</td>
<td>0.98</td>
<td>1.28</td>
<td>1.199</td>
</tr>
<tr>
<td>(110)–LiMnO$_2$</td>
<td>0.88</td>
<td>0.99</td>
<td>1.39</td>
<td>1.233</td>
</tr>
<tr>
<td>(110)–MnO$_2$</td>
<td>0.85</td>
<td>1.19</td>
<td>1.52</td>
<td>1.162</td>
</tr>
<tr>
<td>(111)–Mn</td>
<td>0.63</td>
<td>0.85</td>
<td>0.834</td>
<td></td>
</tr>
<tr>
<td>(111)–Mn$_3$</td>
<td>1.98</td>
<td>1.29</td>
<td>1.18</td>
<td>1.682</td>
</tr>
<tr>
<td>(111)–O$_4$</td>
<td>1.31</td>
<td>1.30</td>
<td>1.201</td>
<td></td>
</tr>
</tbody>
</table>

### Table 6. Formation Energies (eV) of Frenkel and Schottky Defects in Spinel LiMn$_2$O$_4$

<table>
<thead>
<tr>
<th>defect</th>
<th>2NNMEAM $+Q_{eq}$</th>
<th>empirical model</th>
<th>DFT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Li Frenkel (isolated/neighboring)</td>
<td>2.58/1.10</td>
<td>1.65</td>
<td>1.85</td>
</tr>
<tr>
<td>Mn Frenkel (isolated/neighboring)</td>
<td>2.39/2.62</td>
<td>6.27</td>
<td></td>
</tr>
<tr>
<td>O Frenkel (isolated/neighboring)</td>
<td>3.73/3.91</td>
<td>6.69</td>
<td></td>
</tr>
<tr>
<td>Schottky</td>
<td>8.56</td>
<td>5.03</td>
<td></td>
</tr>
</tbody>
</table>

\textsuperscript{a}Reference 4. \textsuperscript{b}Reference 46.
manganese oxide compounds as a function of temperature. The energy of all compounds monotonically increases during heating and recovers to the initial energy after rapid cooling to 0 K, which indicates that the present ternary potential is reliable for dynamic simulations in a wide temperature range and does not cause undesirable phase transformations (the appearance of an unknown structure as a stable phase).

4. DIFFUSION PROPERTIES OF LITHIUM IN SPINEL $\text{Li}_1x\text{Mn}_2\text{O}_4$

As mentioned previously, diffusion properties have a significant influence on the performance of LIB cathode materials. However, it has been restricted to measure the Li diffusion coefficient ($D_{Li}$) experimentally due to the nature of the very light element and no appropriate radioisotope for Li. In addition, the effect of the stoichiometry on the diffusion properties during the charge/discharge process, in which the

Figure 2. Change in energy of lithium manganese oxide phases as a function of temperature (solid symbols, during heating; open symbols, after rapid cooling to 0 K).

Figure 3. (a) Energy of spinel $\text{Li}_1x\text{Mn}_2\text{O}_4$ as a function of temperature and mean-squared displacement (log scale) of lithium atoms in $\text{Li}_1x\text{Mn}_2\text{O}_4$ as a function of time (log scale), where (b) $x = 0$, (c) $x = 0.02$, (d) $x = 0.1$, (e) $x = 0.3$, (f) $x = 0.5$, (g) $x = 0.7$, and (h) $x = 0.9$. 

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lithium vacancy concentration varies by the delithiation/ lithiation, is not well-known experimentally and theoretically. Thus, we investigate the lithium diffusion properties in spinel Li$_{1-x}$Mn$_2$O$_4$ phases through MD simulations. We prepare a pristine spinel LiMn$_2$O$_4$ sample and randomly remove lithium atoms to obtain defective Li$_{1-x}$Mn$_2$O$_4$ samples containing about 1500 atoms, where $x = 0.02, 0.1, 0.3, 0.5, 0.7$, and $0.9$. MD simulations are carried out at different temperatures (300–1400 K) for 10 ps for equilibration and another 1000 ps for simulations. Figure 3a shows the change in internal energy of the Li$_{1-x}$Mn$_2$O$_4$ sample as a function of temperature during the simulations. The LiMn$_2$O$_4$ phase maintains its structure up to 1400 K, as shown in Figures 3a and 2, and the temperature limit at which the structure of the defective Li$_{1-x}$Mn$_2$O$_4$ phase is maintained decreases as the lithium vacancy concentration increases.

We gather trajectories of samples at every 0.1 ps and calculate the mean-squared displacement (MSD) as

$$\text{MSD} = \frac{1}{N} \sum_{i=1}^{N} \langle (r_i(t) - r_i(0))^2 \rangle$$

where $r_i(t)$ is the position vector of the lithium atom $i$ at time $t$, $N$ is the number of lithium atoms, and the angled brackets represent an ensemble average. To reduce statistical errors, the data for 1000 ps is divided into 10 parts, and the MSD values for individual 100 ps are averaged. We calculate the MSD of lithium atoms for each Li$_{1-x}$Mn$_2$O$_4$ sample ($x = 0, 0.02, 0.1, 0.3, 0.5, 0.7$ and $0.9$) in a temperature range in which the structure is maintained. Figure 3b-h shows the calculated MSD as a function of time. The MSD curve is flat at temperatures below 600 K, because it is difficult to observe noticeable diffusion at low temperatures within a conventional MD simulation time scale. At temperatures above 600 K, the MSD linearly increases with time, which indicates the occurrence of lithium diffusion. As the temperature and the lithium vacancy concentration increase, the MSD increases more steeply, indicating that lithium atoms diffuse more easily.

For more quantitative analysis, we calculate the diffusion coefficient ($D$) of lithium using Einstein’s relation

$$D = \frac{1}{2d} \lim_{t \to \infty} \frac{\text{MSD}}{t}$$

where $d$ is the dimension. In the spinel structure, lithium atoms can migrate through three-dimensional channels, and $d$ is therefore 3. Figure 4 shows the calculated $D_{Li}$ of Li$_{1-x}$Mn$_2$O$_4$ in the form of an Arrhenius plot. At low lithium vacancy concentrations, i.e., $x = 0$ and 0.02, the data points tend to deviate from a linear tendency, which is interpreted as a statistical error due to the lack of diffusion sites. As the lithium vacancy concentration increases, the data points are consistent with the linear tendency. At low temperatures below 600 K, the MSD curve is flat, as already mentioned, and $D_{Li}$ cannot be obtained using eq 6. $D_{Li}$ at an ambient temperature can be approximated by extrapolating high-temperature data. The estimated values for $D_{Li}$ of spinel Li$_{1-x}$Mn$_2$O$_4$ at 300 K are within the range of $10^{-18}$–$10^{-16}$ m$^2$/s, as shown in Figure 4, where $x$ varies from 0 to 0.9. The present calculation is in reasonable agreement with the experimental value$^{43}$, 1.3 $\times$ $10^{-18}$ m$^2$/s, obtained by the neutron radiography technique for LiMn$_2$O$_4$ samples.

From the temperature dependency of the calculated $D_{Li}$, the activation energy ($E_a$) of lithium diffusion can be obtained through the following Arrhenius equation

$$D(T) = A \exp \left( \frac{E_a}{k_B T} \right)$$

where $T$ is the temperature, $A$ is the pre-exponential factor, and $k_B$ is the Boltzmann constant. Figure 5 shows the activation energy calculated as a function of the lithium vacancy concentration. Although data for low vacancy concentrations have a relatively high error range due to the statistical error in $D_{Li}$, the calculated values are in a range between 0.50 and 0.64 eV, which is in good agreement with the experimental values of 0.52–1.11 eV determined by Takai et al.$^{43}$ and 0.5 eV determined by Verhoeven et al.$^{45}$ One can expect that the activation energy obtained from the finite-temperature MD simulations may be lower than the migration energy barrier calculated at 0 K, since thermal fluctuations and vacancies in the lattice structure would facilitate the migration of lithium atoms. Indeed, the migration energy barrier obtained from the finite-temperature MD simulations, 0.50–0.64 eV, is slightly lower than the 0 K value, 0.7 eV.

The calculated diffusion properties are self-consistent (showing a reasonable difference between finite-temperature values and the 0 K value of the migration energy barrier) and are in good agreement with the experimental data. We have found that the effect of lithium vacancy concentration is marginal on the activation energy for lithium diffusion in the Li$_{1-x}$Mn$_2$O$_4$ cathode, but it is significant on the lithium diffusion coefficient. However, it is necessary to mention about whether
our model describes the lithium diffusion based on an accurate physical principal. It is known that the lithium migration in oxide lattice structures is strongly influenced by a correlation effect of lithium ion and electron polaron.66,65 Our model does not exclusively account for the polaron effect, and therefore, it may not be able to correctly reproduce the lithium diffusion properties. However, our model is based on a variable charge scheme, in which the lithium charge state is changed by the surrounding atomic environment as the lithium atom migrates. We hope and believe that the polaron effect is not completely neglected in our model.

Our ultimate goal is to analyze the effect of Mn, Co, and Ni composition on the lithium diffusion property, and we are developing Li–Co–O and Li–Ni–O potentials in a similar way. We believe that the approximation made for the polaron effect is applied similarly to each system, so that the potentials can be sufficiently utilized for large-scale atomistic simulations to predict the diffusion, phase transition, and defect properties of Li–(Co, Mn, Ni)–O multicomponent systems under various conditions of structure, composition, and alloying elements. The diffusion simulations described above can be carried out for Li–(Co, Mn, Ni)–O multicomponent systems in the iBat simulation platform (http://battery.vfab.org). iBat also provides various multiscale simulation techniques for LIB materials.

5. CONCLUSION

It has been shown that the 2NNMEAM+Qeq interatomic potential developed for the ternary Li–Mn–O system can describe various fundamental physical properties of lithium oxides, manganese oxides, and lithium manganese ternary oxides in reasonable agreement with relevant experimental data and/or DFT calculations. The lithium diffusion properties predicted by MD simulations based on this potential are self-consistent and match reasonably well with experiments. It is found that the lithium vacancy concentration has a significant influence on the lithium diffusion coefficient, while it has only marginal influence the activation energy for lithium diffusion. The developed Li–Mn–O potential can also be utilized for atomic simulations on LIB cathode materials properties such as phase transitions, defect formation, and lithiation/delithiation. In addition, the potential can be further extended to multicomponent systems by combination with potentials for other Li–TM–O (TM: Co, Ni, etc.) systems and can be utilized for atomistic simulations on a wider range of LIB cathode materials.

ASSOCIATED CONTENT

Supporting Information
The Supporting Information is available free of charge on the ACS Publications website at DOI: 10.1021/acs.jpcc.7b02727.

Formalism of the 2NNMEAM and 2NNMEAM+Qeq interatomic potential models and the evaluation of binary potentials (Li–Mn, Li–O, and Mn–O) (PDF)

Source code for molecular dynamics simulation based on the 2NNMEAM+Qeq potential (ZIP)

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The authors declare no competing financial interest.

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